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The Keys of Atlantis

A Study of Ancient Unified Numerical and Metrological Systems

by Peter Wakefield Sault

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The Keys of Atlantis

Chapter 3.

Mountains of The Moon

The Stone-Cored Pyramids of Egypt

PROLEGOMENON

This little treatise is a mathematical, metrological, geographical and astronomical analysis of certain Egyptian pyramids. English translations of their ancient traditional names, less the names of the kings to whom rightly or wrongly a much later Egyptian priesthood attributed their building, are used here throughout except when quoting other authors. This is problematic since the original Great Pyramid is nowadays generally referred to as Khafre, Khephren or the Second Giza Pyramid. The original name of the pyramid now widely though erroneously known as the 'Great Pyramid' is Pyramid of The Horizon, or simply Horizon Pyramid. If, that is to say, 'Horizon' is the correct interpretation of the hieroglyph *Akhet*, about which some apparently justifiable doubt remains. In order to minimise the reader's possible confusion, the original and true Great Pyramid (i.e. 'Khephren') will here be referred to as the Grand Pyramid.

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3-1. The Giza Complex



Figure 3-1. The Pyramids of Giza (1)

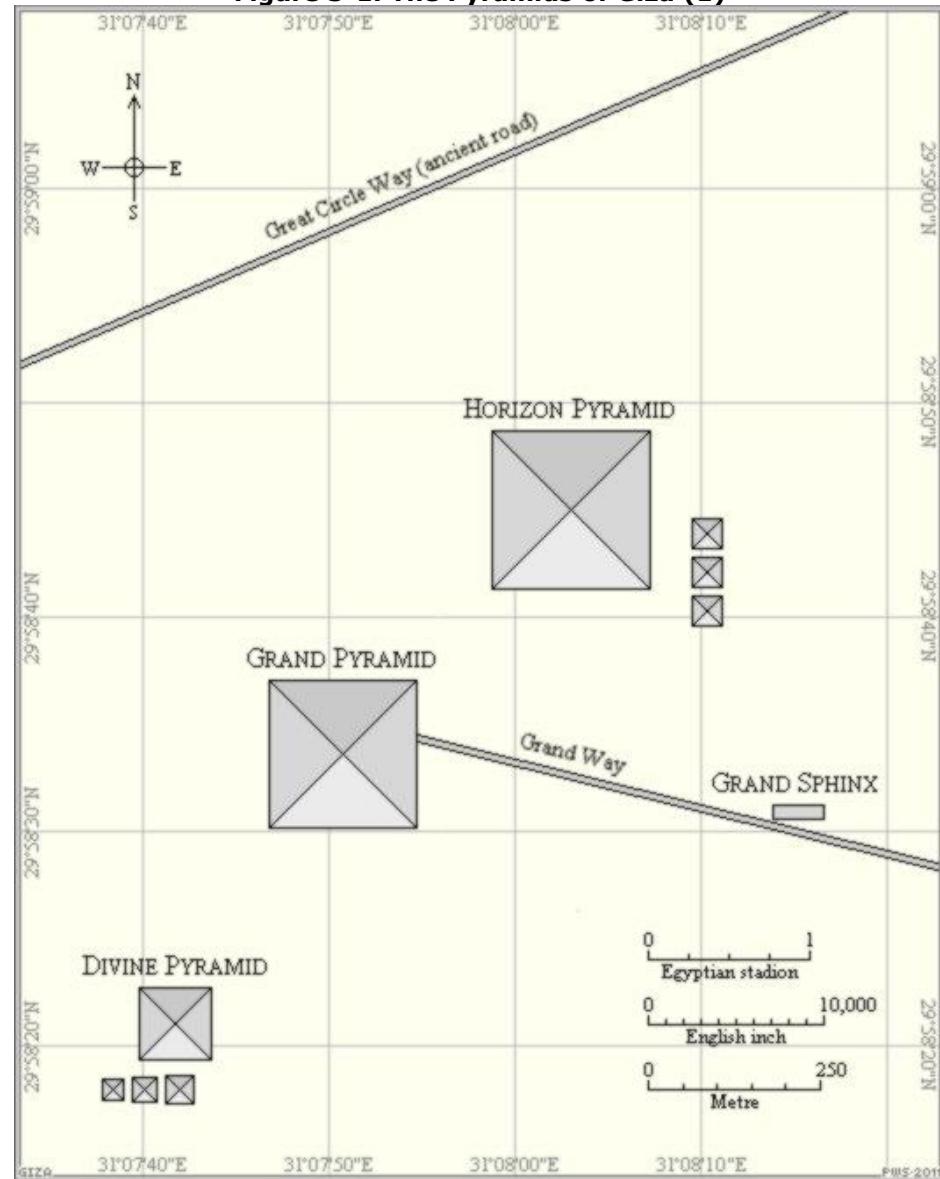


Figure 3-2. Map of The Giza Pyramid Complex

3-2. The Horizon Pyramid

The Pyramid of The Horizon, known variously although erroneously as the Pyramid of Khufu, Pyramid of Kheops, Great Pyramid or First Giza Pyramid, stands on the Giza Plateau in Egypt at $29^{\circ} 58' 45''$ N $\times 31^{\circ} 08' 03''$ E. These and all other geographical coordinates given in this chapter have been obtained since 2006 with the aid of *Google Earth* and supercede all earlier figures from other sources.

Peter Tompkins summarizes the origin of the Horizon Pyramid thus[1]

"Though all agree that the Great [Horizon] Pyramid is at least four thousand years old, none can say for certain just when it was built, by whom, or why."

The first thing to note about this observation is that Tompkins refrains from attempting to assign a *maximum* age to the Horizon Pyramid.

Tompkins' book 'Secrets of The Great [Horizon] Pyramid' (1971) is without doubt the most comprehensive compendium of myths, legends and knowledge about the Horizon Pyramid ever published and was, as noted on his website*, "not yet faulted in its original content". Until now, that is to say, firstly in respect of his misapplication of the name 'Great' to the Horizon Pyramid and secondly in his treatment of Isaac Newton's fictive 'Memphis Cubit' as if it were a genuine

unit of measure when it was never more than a figment of Newton's fevered imagination. These matters will be discussed in detail in the final section of this chapter, 'The Egyptological Fraud'.

Tompkins goes on to say[2]

"It would be satisfactory to be able to describe the method by which the Great [Horizon] Pyramid was put together, by whom, and when."

"But the builders, whoever they may have been, left no description of their method. No one has even found a *later* Egyptian report of how the first pyramids were built."

The earliest extant reference to the Horizon Pyramid comes from Herodotus, who says[3]

"To build the pyramid itself took twenty years; it is square at the base, its height (800 feet) equal to the length of each side; it is of polished stone blocks beautifully fitted, none of the blocks being less than thirty feet long."

It would be another 2,000 years before the Horizon Pyramid was accurately surveyed and the relationship between its height and the length of each side thereby properly observed. With the missing part of the statement inserted:- *The quadrant whose radius is the height is equal to the length of each side.*

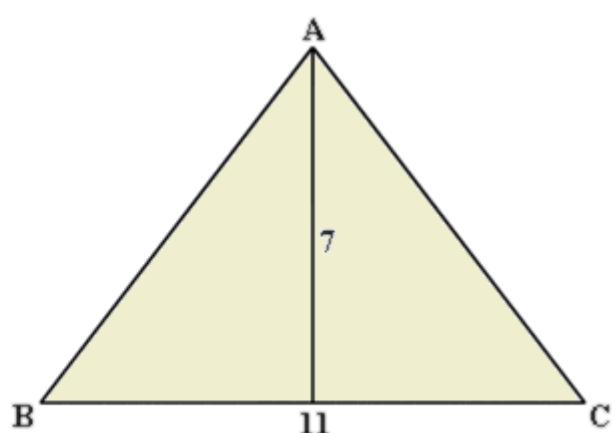
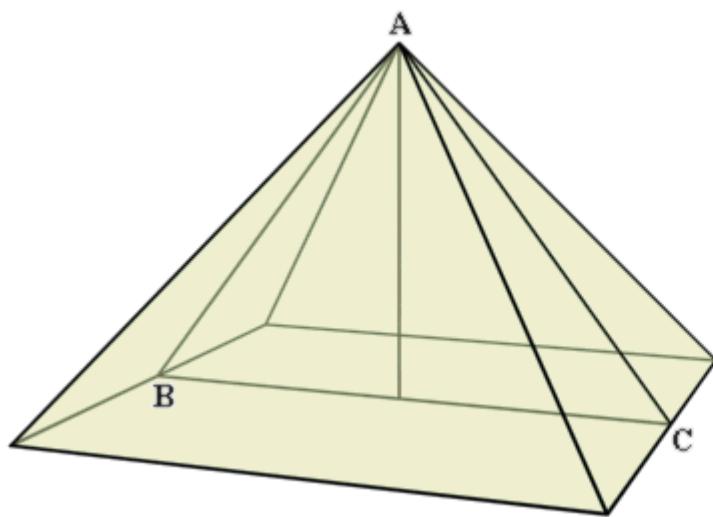


Figure 3-3. Theoretical Proportions of the Horizon Pyramid

Some of the earlier surveys contained errors and gave rise to some preposterous theories, none of which will be discussed here, about which Prof. W.M.F.Petrie says[8]

"It is useless to state the real truth of the matter, as it has no effect on those who are subject to this type of hallucination. They can but be left with the flat earth believers and other such people to whom a theory is dearer than a fact."

The results of the two most reputable surveys of the Giza pyramids, those of W.M.F.Petrie (1881) and J.H.Cole (1925, Horizon Pyramid only), are shown in [Appendix C](#). Since these surveys were conducted,

certain peculiarities in the form of the Horizon Pyramid have come to light, not the least being a very slight indentation of the sides which results in the distance across the middle of the base (BC in Figure 3-3), between opposite apothems (AB and AC in Figure 3-3), being shorter than the lengths of the sides parallel to it, by about 1.2 metres. The indentations went unnoticed until discovered by aerial photography and are visible only when the Sun is shining across a face, throwing half of that face into shadow. Their presence testifies to the high degree of precision with which the Horizon Pyramid was built.

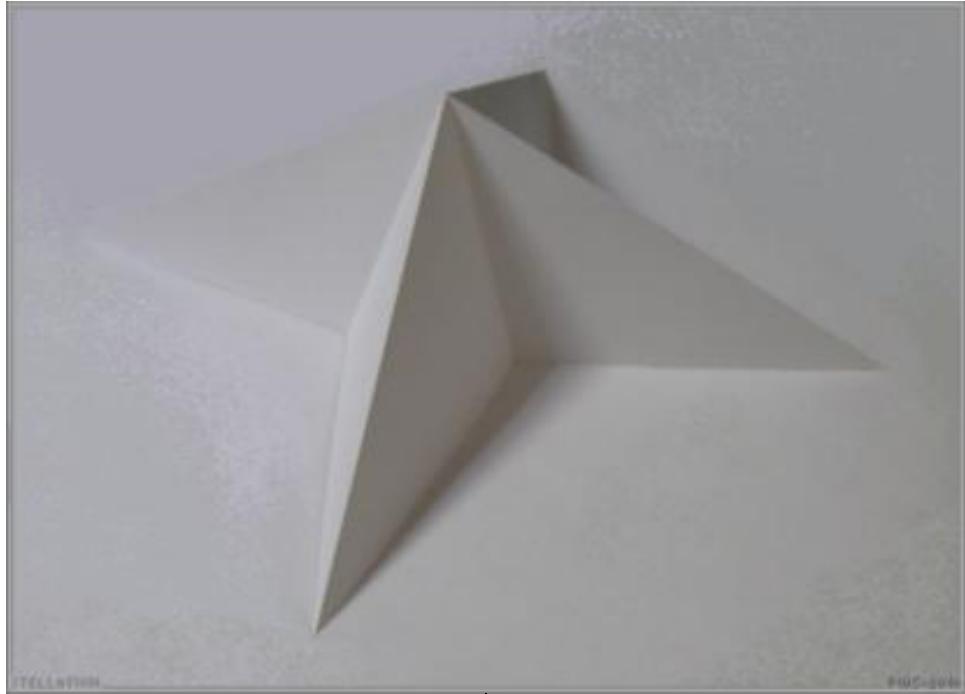


Figure 3-4. Stellated Pyramid

The first person to comment on the apparent π (pi) relationship between the height of the Pyramid and the perimeter of its base was John Taylor, working from measurements made by John Perring during Howard Vyse's expedition in the 1830s. Petrie later opined that it formed the basis of one of the best theories regarding the intended dimensions. Of the angle and elevation of the Horizon Pyramid, Petrie says[12]

"On the whole, we probably cannot do better than take $51^{\circ}52' \pm 2'$ as the nearest approximation to the mean angle of the Pyramid, allowing some weight to the South side.

"The mean base being $9,068.8 \pm 0.5$ inches, this yields a height of $5,776.0 \pm 7.0$ inches."

This gives a range of ratios of base-width to height, varying between $(9068.8 - 0.5)/(5776.0 + 7.0)$ and $(9068.8 + 0.5)/(5776.0 - 7.0)$, or 1.5681 and 1.5721, with a difference of 0.004, or roughly 0.25%. It can be seen that the values $\pi/2$ (~ 1.5708) and $11/7$ (~ 1.5714) both fall within this range.

This means that, whatever the true or intended dimensions, the height is equal to the radius of a circle whose circumference very closely approximates the perimeter of the base.

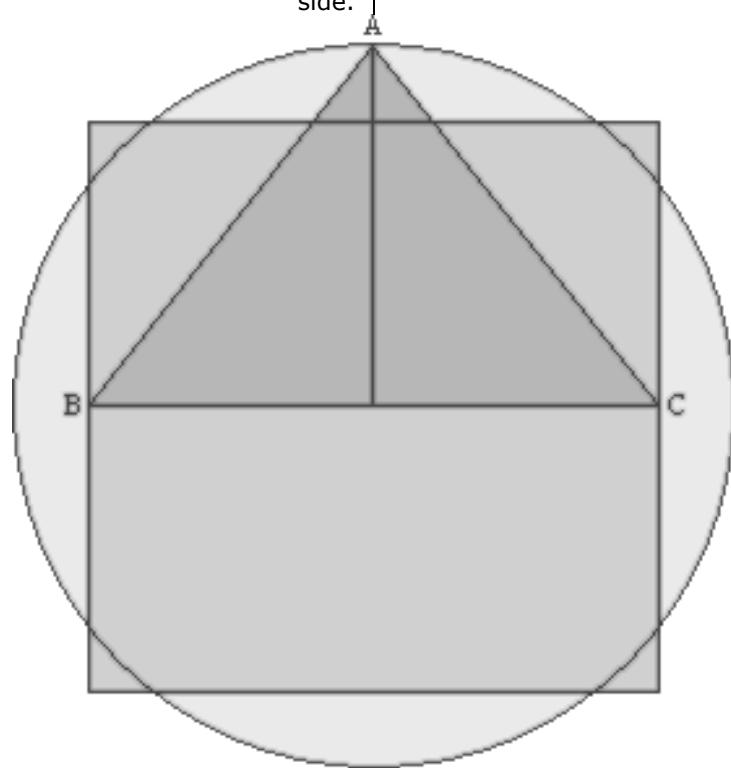


Figure 3-5. Squaring The Circle – Geometry of The Horizon Pyramid

The attribution of the building of the Horizon Pyramid to Khufu was documented first by Herodotus, who took it

as fact on the basis of assertions made by the priests of Sais.

The modern evidence is equally suspect; some red ochre paint daubed on a block of stone in one of the cavities above the metrological cavity, the so-called King's Chamber, allegedly showing the cartouche of Khufu. There were, however, rumours of paintbrushes and cans of red ochre paint, still used by Egyptian quarry workers nowadays to mark stones, being carried into the Horizon Pyramid during Howard Vyse's expedition coincident with Vyse's need to raise new funding. This he succeeded in doing following the announcement of his convenient 'discovery' that the Horizon Pyramid could indeed be attributed to Khufu and therefore confidently dated – happily for everyone – in accordance with both Egyptian kinglists and the Torah's dating of 'Creation'.

Moreover, the ancient cemeteries and boat pits adjacent to the Horizon Pyramid seem unlikely to have belonged to the builders or original keepers of any part of the Giza complex since they encroach upon the Horizon Pyramid's precinct, as can be seen quite clearly in satellite images. Such travesties would simply never have been permitted by the builders since all they have ever done is to despoil and desecrate the complex, as can also be seen quite clearly in satellite images. The later pyramid cults to which the cemeteries bear witness appear to have had as much to do with the building of the Giza pyramids as latterday ersatz 'druids' have to do with that of Stonehenge. The remains of the precinct walls of the two other main Giza pyramids can also clearly be seen in satellite images.

* * *

The first person known to have looked for a linear modulus of the Horizon Pyramid in the dimensions of the metrological cavity was Sir Isaac Newton (1642-1727), working from measurements of it made separately by John Greaves (1638) and Tito Livio Burattini (1642). Through a particularly abstruse line of argument, Newton concluded that this modulus was equal to a tenth part of the width of the cavity. It was the dimensions of the Earth that Newton was looking for in order to confirm his theory of gravitation and he had acquired from an unknown source the belief that the builders had encoded these dimensions in those of the Horizon Pyramid. As Tompkins records[3]

"...it was from Greaves's data that Sir Isaac Newton deduced that the Great [Horizon] Pyramid had been built on the basis of two different cubits, one of which he called 'profane' and the other which called 'sacred'. From Greaves's and Burattini's measurements of the King's Chamber, Newton computed that a cubit of 20.63 British inches produced a room with an even length of cubits: 20 x 10. This cubit Newton called the 'profane', or 'Memphis', cubit; whereas a longer, more arcane cubit appeared to measure about 25 British inches.

"This longer, or 'sacred', cubit Newton derived from the Jewish historian Josephus's description of the circumference of the pillars of the Temple at Jerusalem. Newton estimated this cubit to be between 24.80 and 25.02 English inches, but believed the figure could be refined through further measurements of the Great [Horizon] Pyramid and other ancient buildings.

"All of this Newton wrote up in a small and now hard-to-find paper called A Dissertation upon the Sacred Cubit of the Jews and the Cubits of several Nations; in

which, as taken by Mr. John Greaves, the ancient Cubit of Memphis is determined.

"Newton's preoccupation with establishing the cubit of the ancient Egyptians was no idle curiosity, nor just a desire to find a universal standard of measure; his general theory of gravitation, which he had not yet announced, was dependent on an accurate knowledge of the circumference of the earth. All he had to go on were the old figures of Eratosthenes and his followers, and on their figures his theory did not work out accurately.

"By establishing the cubit of the ancient Egyptians, Newton hoped to find the exact length of their stadium, reputed by classical authors to bear a relation to a geographical degree, and this he believed to be somehow enshrined in the proportions of the Great [Horizon] Pyramid."

Newton was working backwards from a purely hypothetical Israelite cubit allegedly described by a now largely discredited historian and of which no physical example existed, via the ratio 6:7 (that of a *hypothetical* 'profane', or 'common', cubit to a *hypothetical* 'sacred', or 'royal', cubit and an approximation to $\sqrt{3}:2$, that being the exact ratio of the diameters of 30th Parallel and Equator on a sphere), to a division by 10 of the width of the floor of the metrological cavity of the Horizon Pyramid thence to postulate a 'Memphis' cubit that had existed since the foundation of Egypt. Clearly this line of argument is far-fetched, to say the very least. Nonetheless, subsequent authorities bowed to that of Newton and, for example, Dr. Charles Piazzi Smyth lauded Newton thus:[4]

"How thankful should we be that it pleased God to raise up the spirit of Newton amongst us; and enable him to make one of the most important discoveries of his riper years – though the opposition of the Church of England has caused it to remain unread almost to the present day – that while there undoubtedly was in ancient times a cubit of 20.7 inches nearly... ...and which Newton calls 'the profane cubit' there was another which he equally unhesitatingly speaks of as the *sacred* cubit, decidedly longer."

Petrie also accepted without question the use of Newton's *Israelite* cubit in the construction of the Horizon Pyramid [9]

"Thus the total length of the plug-blocks would be about 203 inches, or very probably 206 inches, or 10 [Newton's] cubits, like so many lengths marked out in that passage."

And then Prof. Livio Stecchini adds his stamp of approval:[6]

"It was Newton who, on the basis of the survey conducted by Greaves, realized that the King's Chamber measures 10 by 20 [Newton's] cubits. Having established this fact,..."

Others, however, have noticed the insubstantial nature of Newton's cubit, Derek Gjertsen observing in respect of it that[14]

"As with many such works, both before and after, however tight the reasoning, the reliability of the conclusions will still depend upon the accuracy of the initial assumptions. As many of these were, in fact, no more than guesses, Newton's conclusions lose much of their initial plausibility."

Despite its dubious pedigree many contemporary authors continue to treat Newton's cubit as if it was genuine.

The only clear facts are that the proportions of the floor of the metrological cavity are 1:2 and that the length is a whole-number division of both the width and the height of the pyramid, making these 22 and 14 cavity-lengths respectively. Hence, for the floor, the longest common linear extent is the width of the cavity, 206.3 ± 0.2 , English inches (5.24m). This is indeed close to Petrie's reported length of the plug-blocks, as noted above. The width of the cavity is to its height as 9:10 and taking this into account the longest common length that can be derived from all three of its dimensions is a unit equal to a tenth part of the *height* of the cavity, again unlike Newton's cubit, at ~ 22.9 English inches (582.2mm). In terms of this linear modulus, or *Horizon cubit*, the width of the base and the height of the Horizon Pyramid come out at 396 and 252 respectively. The figure for the width of the base is remarkably close to that suggested by French scientist Edmé-François Jomard (1777-1862), 400 *pyk belady*, that being the number of cubits to a stade.^[5]

"In vain Jomard argued that he had found an even more surprising coincidence in that the four-hundredth part of his base of the [Horizon] Pyramid gave a figure of .5773 meter, which was exactly the length of a longer modern Egyptian cubit called the *pyk belady*."

Jomard's measure of the width of the base was, however, slightly excessive since 1/400th of the actual width is 0.5759 metre (22.67 inches), taking J.H.Cole's mean value of 230.364 metres, where 1/396th gives 0.5817 metre (22.9 inches). Moreover, the square stade is the area of the *sacred precinct*, not of the temple which stands within it. Hence the Horizon stade would be expected to exceed the width of the Horizon Pyramid. Four hundred Horizon cubits gives a stade of some 9,160 inches (232.66 metre). Curiously, 1/400th of Perring's erroneous figure for the width of the base, 764 English feet, is the *pyk belady* of 22.9 English inches.

Although the *pyk belady* was in use in Egypt in Jomard's time nothing is known of its origin and history therefore no firm conclusion can be reached about the age of the Horizon Pyramid from the fact that it is the modulus. All that can be said of the modulus is that it must of necessity predate any structure which incorporates it. The name '*pyk belady*' combines the Egyptian word for a cubit, related to the Greek PHCUS, *pekhos* and PUGWN, *pygon*, both also meaning a cubit, with that for 'our land', as in the modern Egyptian national anthem 'Belady, Belady, Belady'. Hence '*pyk belady*' can be taken to mean 'national cubit [of Egypt]'.

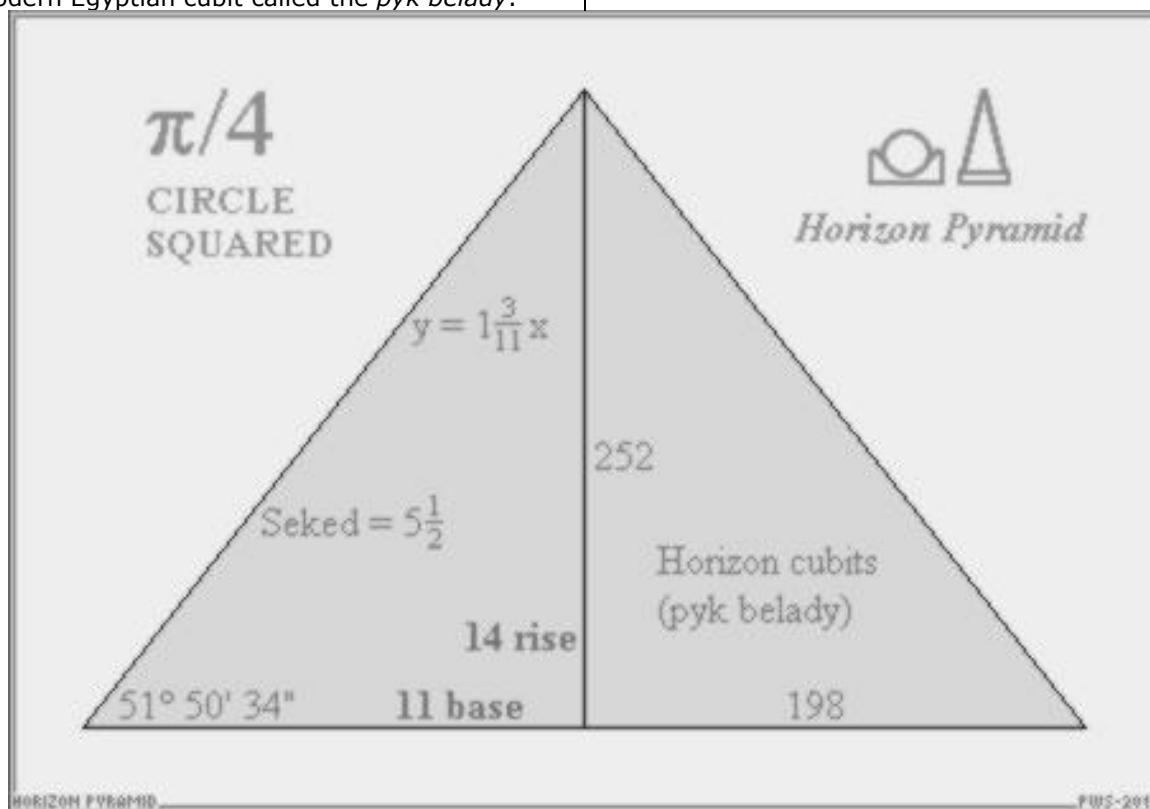


Figure 3-6. Theoretical Cross-Section of The Horizon Pyramid

It might be thought that metrological standards in ancient times followed the same principle as those of today and that, therefore, a given unit would have measured exactly the same absolute quantity no matter what the context. However, as was seen in the case of the Parthenon stade, the unit used for the temple, although approximating it, was not the same as that of the general tradesman. There seems to be no intentional conversion ratio between the two standards,

which merely approximate one another, varying only by a few inches; varying nonetheless. It would appear that in the field of monumental architecture units were consistent only within the context of a particular undertaking and that an architectural guild was at liberty to contrive its own linear standards project by project within a fairly narrow range of lengths as long as they conformed to a uniform fixed system. The particular set of standards for each pyramid was

created and ritually consecrated specifically for that pyramid then ritually desecrated and made useless once the building was complete. A remnant of this ritual destruction of the project standards can be seen in the modern ceremony of *cutting the ribbon* at the official opening of a new public building by some or other dignitary. The ribbon symbolises the primary standard, the stade rope, used in the construction.

The averaged, corrected measures of the metrological cavity are shown in Table 3-1. Petrie's words concerning the condition of the cavity as he found it must be borne in mind.[\[10\]](#)

"The King's Chamber was more completely measured than any other part of the Pyramid; the distances of the walls apart, their verticality in each corner, the course heights, and the levels, were completely observed. On every side the joints of the stones have separated, and the whole chamber is shaken larger. By examining the joints all round the 2nd course, the sum of the estimated openings is, 3 joints opened on N. side, total = .19 [English inches]; 1 joint on E. = .14; 5 joints on S. = .41; 2 joints on W. = .38. And these quantities must be deducted from the measures, in order to get the true original lengths of the chamber. I also observed, in measuring the top near the W., that the width from N. to S. is lengthened .3 by a crack at the S. side.

"These openings or cracks are but the milder signs of the great injury that that the whole chamber has sustained, probably by an earthquake, when every roof beam was broken across near the South side; and since which the whole of the granite ceiling (weighing some 400 tons), is upheld solely by sticking and thrusting. Not only has this wreck overtaken the chamber itself, but in every one of the spaces above it are the massive roof-beams either cracked or torn out of the wall, more or less, at the South side; and the great Eastern and Western walls of limestone, between, and independent of which, the whole of these construction chambers are built, have sunk bodily. All these motions are yet but small - only a matter of an inch or two - but enough to wreck the theoretical strength and stability of these chambers, and to make their downfall a mere question of time and earthquakes."

Dimension	English Inches	Metres	Horizon Cubits
Length	412.25	10.4712	18
Width	206.13	5.2357	9
Height	229.??	5.82??	10
Total	847.38	21.5387	37
Mean	22.90	0.5822	1

Table 3-1. Theoretical Dimensions of The Metrological Cavity of The Horizon Pyramid

It is worth noting Petrie's comment about the levels of floor and ceiling[11].

"The floor of the chamber, as is well known, is quite disconnected from the walls, and stands somewhat above the base of the lowest course. It is very irregular

in its level, not only absolutely, but even in relation to the courses; its depth below the first course joint varying 2.29 [English inches], from 42.94 to 40.65." This is an example of the marriage of certain irrational and mutually incommensurable numerical constants, such as π (pi) and $\sqrt{2}$ (square root of 2), using a system of whole numbers and their ratios. In order to do this, some rough-and-ready approximations must be incorporated. The primary approximation expressed by the gross form of the Horizon Pyramid is that of $6\frac{2}{7}$, or the perimeter of the base divided by the height, as a representation of the value 2π . Hence there can be no exact expression of any of the constants referenced in any of the physical approximations embodied in any of the structures. In fact, to find a single exact value anywhere would be to exclude all possibility of finding comparable references. The architect has created a compendium of equally variant *Diophantine* approximations.

In terms of the proportion 9:10:18, employing the derived Horizon cubit of ~22.9 English inches:- The theoretical surface area of the metrological cavity is 864 square Horizon cubits. This is equal to the surface area of a cube of edge 12 Horizon cubits. The volumes of the cavity and its cube of equal surface area are in the ratio of a naturally tempered musical *semitone*, 15:16. This is also the ratio of both the widths of the bases (variance 0.34%) and the capacities of the coffers of the Grand and Horizon Pyramids. The sum of the lengths of all the edges of a 12-cubit cube is to that of the cavity as 36:37. The theoretical volume of the Horizon Pyramid is 13,172,544 cubic Horizon cubits.

$$13,172,544 = 2^6 \times 3^5 \times 7^1 \times 11^2 = 7 \times 33^2 \times 12^3$$

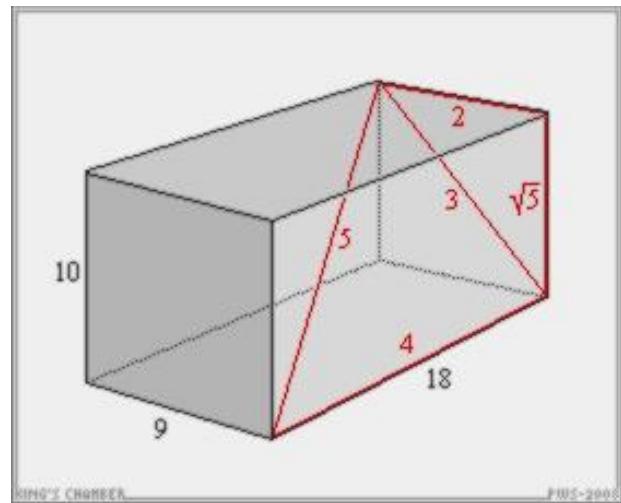


Figure 3-7. Theoretical Proportions of The Metrological Cavity of The Horizon Pyramid

That is exactly equal to 7,623 12-cubit cubes; cubes, that is, of surface area equal to that of the theoretical metrological cavity. Now, these 7,623 cubes can be arranged as a stack of 7 equal square tiers of width 33. With the width of the tiers equal to the width of the base of the pyramid, the height of the resulting rectangular block is exactly 1/3rd of the height of the pyramid. Altogether, the figure clearly embodies in whole numbers the arithmetical method for calculating

the volume of any pyramid; one third of the height times the area of the base.

Hence it may be concluded that the Horizon Pyramid was laid out and constructed using the *pyk belady* as its modulus. However, because nothing is known about the history of this unit of measure beyond its use in the construction of the Horizon Pyramid and its general use in Egypt at the time of Jomard's visit, its age and origin cannot be established and, therefore, no firm conclusion can be drawn about the age of the Horizon Pyramid from its use. Jomard's observation is seemingly the only connection to any memory of its existence, since all other mentions of this unit themselves refer back to him as their source. Nonetheless it is clear that the linear modulus of the Horizon Pyramid possesses a unique name and is no mere hypothesis. Not only that but the Horizon Pyramid linear modulus can confidently be labelled the *Egyptian cubit*, for that indeed is what it is.

Taking the disk of the Earth as a circle and the value of pi as 22/7, in Egyptian stades of 400 cubits the circumference so obtained is exactly 171,072 Egyptian stades and the radius 27,216. A [Greek stade](#) on the same circle would be exactly equal to 99/125 Egyptian stade (terrestrial circumference = 216,000 Greek stades).

The Horizon Pyramid width, at 99/100 Egyptian stades, is equally interesting since the terrestrial circumference comes out at 172,800 or 100 times the cube of 12. Moreover, again on the same circle, it is exactly equal to 1 1/4 Greek stades, the Greek metric standards having been supplied to Solon by the Temple of Neith at Sais around 600 BC, suggesting an unbroken tradition stretching from the building of the Giza complex to that of the Parthenon of Athens. The Greek system of linear measures was derived from the Egyptian and both include a digit (finger) as the smallest unit, a foot of 16 digits, a cubit of 24 digits and a stade of 600 feet.

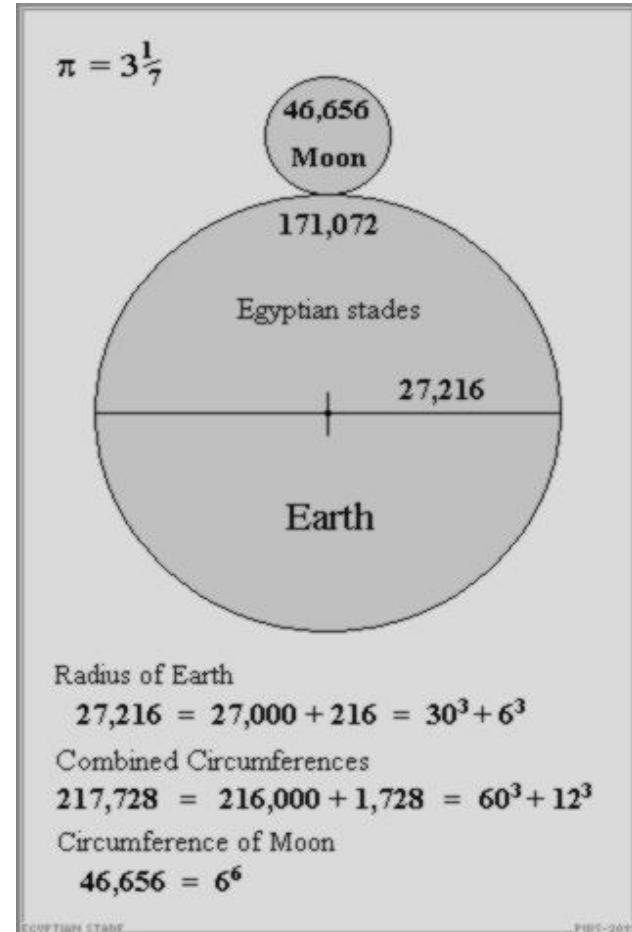


Figure 3-8. The Egyptian Stade

* [Peter Tompkins' website](#) was removed from the World-Wide Web in February 2008, a year after his death in January 2007. This author has a copy saved in his archives.

3-3. The Grand Pyramid

The mean width of the base of the Grand Pyramid is almost exactly 370 Horizon cubits (*pyk belady*), the variance being roughly 1 part in 6,000 or 0.016%, and here the rounded figure will be assumed to be the intended width. This pyramid was apparently intended to conform to the proportions of a Pythagorean {3-4-5} triangle, giving a theoretical height of $246 \frac{2}{3}$ cubits or 370 feet.

The theoretical volume of the Grand Pyramid is exactly 37,989,750 cubic Horizon feet.

$$37,989,750 = 2 \times 3 \times 5^3 \times 37^3 = 6 \times 185^3$$

The length of each edge of the 6 cubes so defined is equal to half the height of, and to one third the width of, the pyramid.



Figure 3-9. Theoretical Cross-Section of The Grand Pyramid

3-4. The Divine Pyramid

Petrie's mean for the width of the Divine Pyramid is 4,153.6 English inches, or roughly 181.34 Horizon cubits with a probable intended figure of $181 \frac{1}{3}$ cubits, or 272 Horizon feet exactly, differing from the mean by -0.004%. For the angle of the apothem Petrie's "best conclusion" is $51^\circ 0' \pm 10'$. This results in a height somewhere between 2549 and 2580 English inches, or roughly 167 and 169 Horizon feet. Taking the mean gives a height of 168 feet and this will be found to share a common factor of 8 with the theoretical width.

Hence the proportions are 21 rise on 17 base, revealing a clearly intended allusion in the division of the width by the height to **F** (Phi), the *Golden Mean* or *Divine Proportion*, differing from it by a mere +0.06%. In fact the ratio of height into width, 21:34, displayed by the Divine Pyramid is the 8th term of the *Fibonacci Sequence*, which converges to Phi.[\[18\]](#)

$$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \dots$$



Figure 3-10. Theoretical Cross-Section of The Divine Pyramid

The theoretical volume of the Divine Pyramid is exactly 4,143,104 cubic Horizon feet.

$$4,143,104 = 7 \times 34^2 \times 8^3$$

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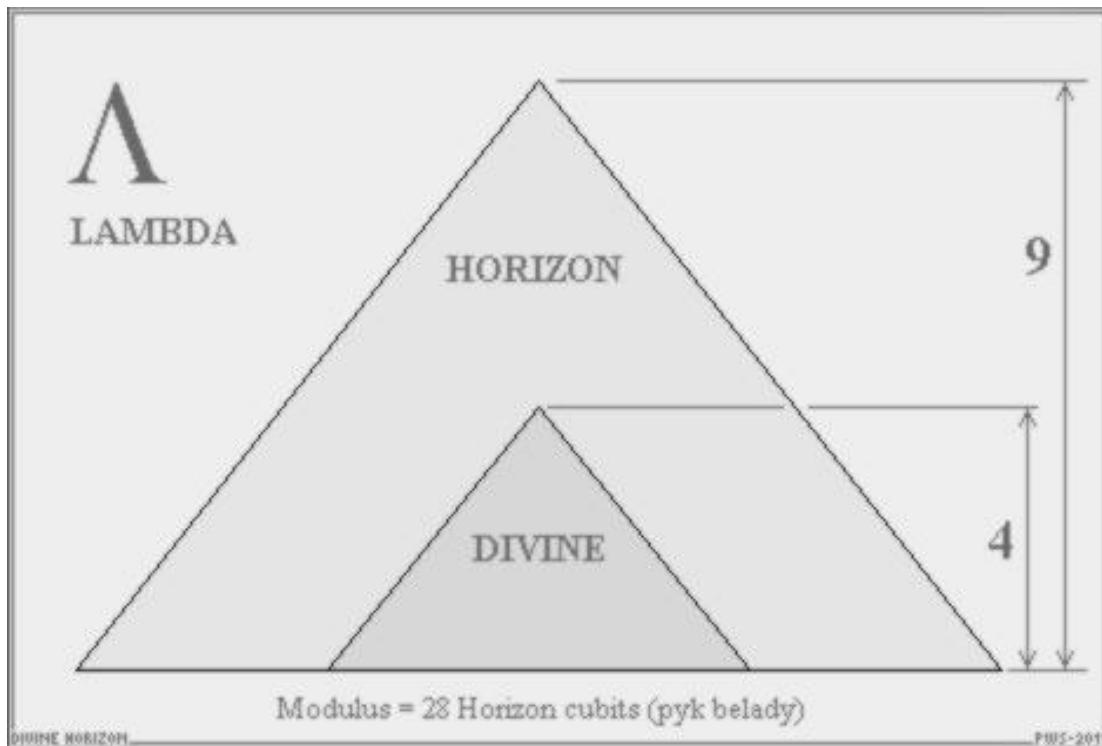


Figure 3-11. The Ratio between The Heights of Horizon And Divine Pyramids

The ratio between the heights of Horizon and Divine pyramids is the same as that between the length and breadth respectively of the platform of the Parthenon of Athens - 4:9. This ratio belongs to the *Lambda*, itself the root and source of the dodekaphonic musical scale,

as described in [Chapter 1](#), suggesting that the Giza complex comprises an integrated design and that its overall planning was completed before a single stone was quarried for a single pyramid.



Figure 3-12. The Pyramids of Giza (2)

3-5. The Southern Shining Pyramid



Figure 3-13. The Southern Shining Pyramid

The reader can forget the Egyptological dogma that seeks to explain the compound slope of the Southern Shining Pyramid as a decision by the builders to switch slopes in mid-project, the rationalization of this usually being that they discovered that the internal stresses of the structure prevented completion of a pyramid with

the continued slope of the lower section. How exactly they discovered this is never made clear and no evidence supporting the stress-factor hypothesis is ever presented. It is of course a flight of fancy, for from the outset the Southern Shining Pyramid was designed to

be a flattened obelisk, as the following analysis of its geometry will make very clear.

Quantity	Measure
Width at ground level	188 metres (617 feet)
Height overall	105 metres (345 feet)
Upper dihedral angle	43° 22'
Lower dihedral angle	54° 27' 44"

Table 3-2. Reported Measures of The Southern Shining Pyramid [16]

As with the Horizon Pyramid, the linear modulus of the Southern Shining Pyramid is a 9th part of the width, 207.09 English inches, of its principal cavity, giving a Southern Shining cubit of 23.01 English inches. With this unit the height of the pyramid emerges as 180 cubits and the width of its base as 324 cubits. The rest follows accordingly as shown in Figure 3-14. A small adjustment to the reported dihedral angle of the upper faces is needed to bring the whole scheme into perfection.

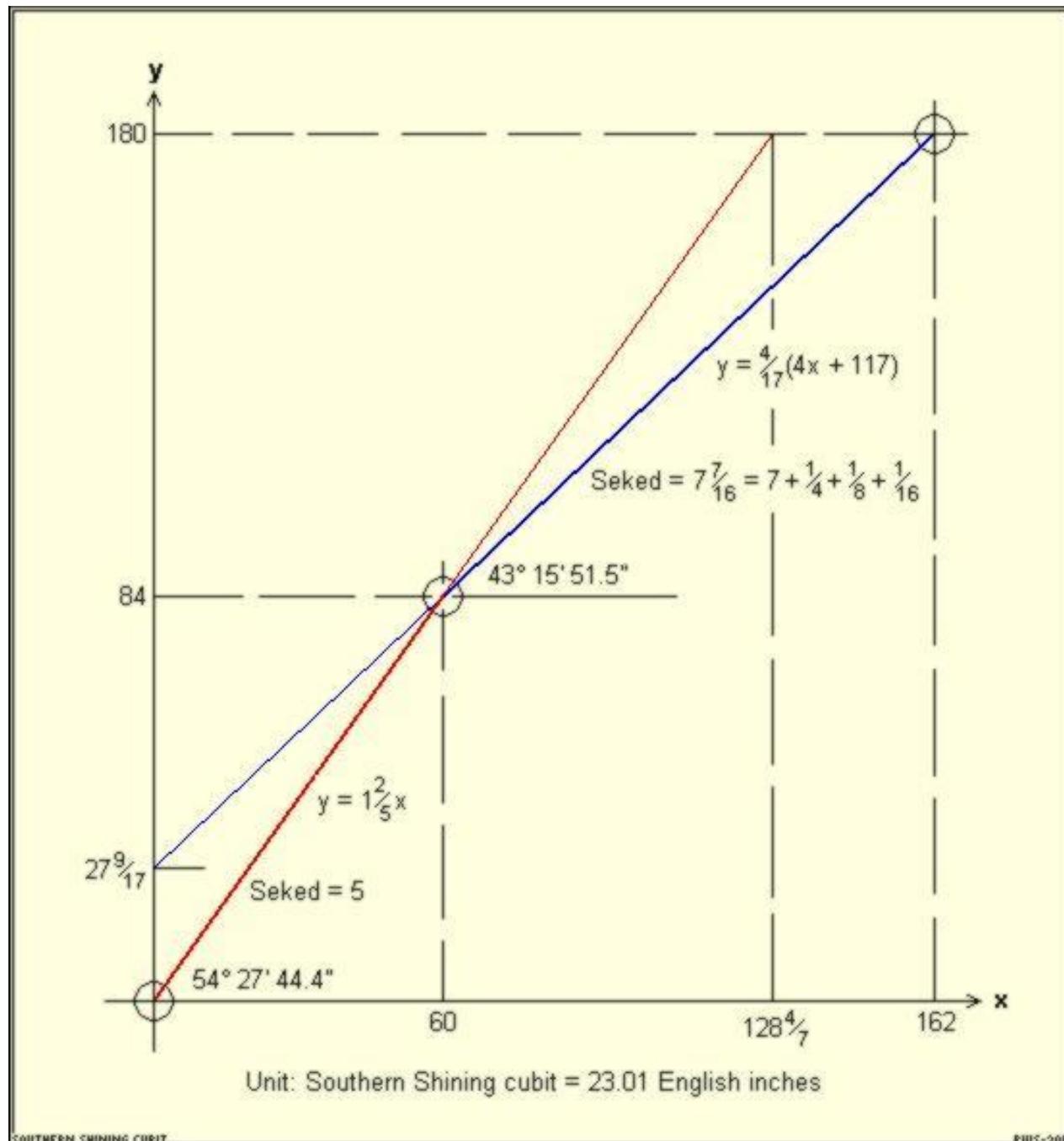
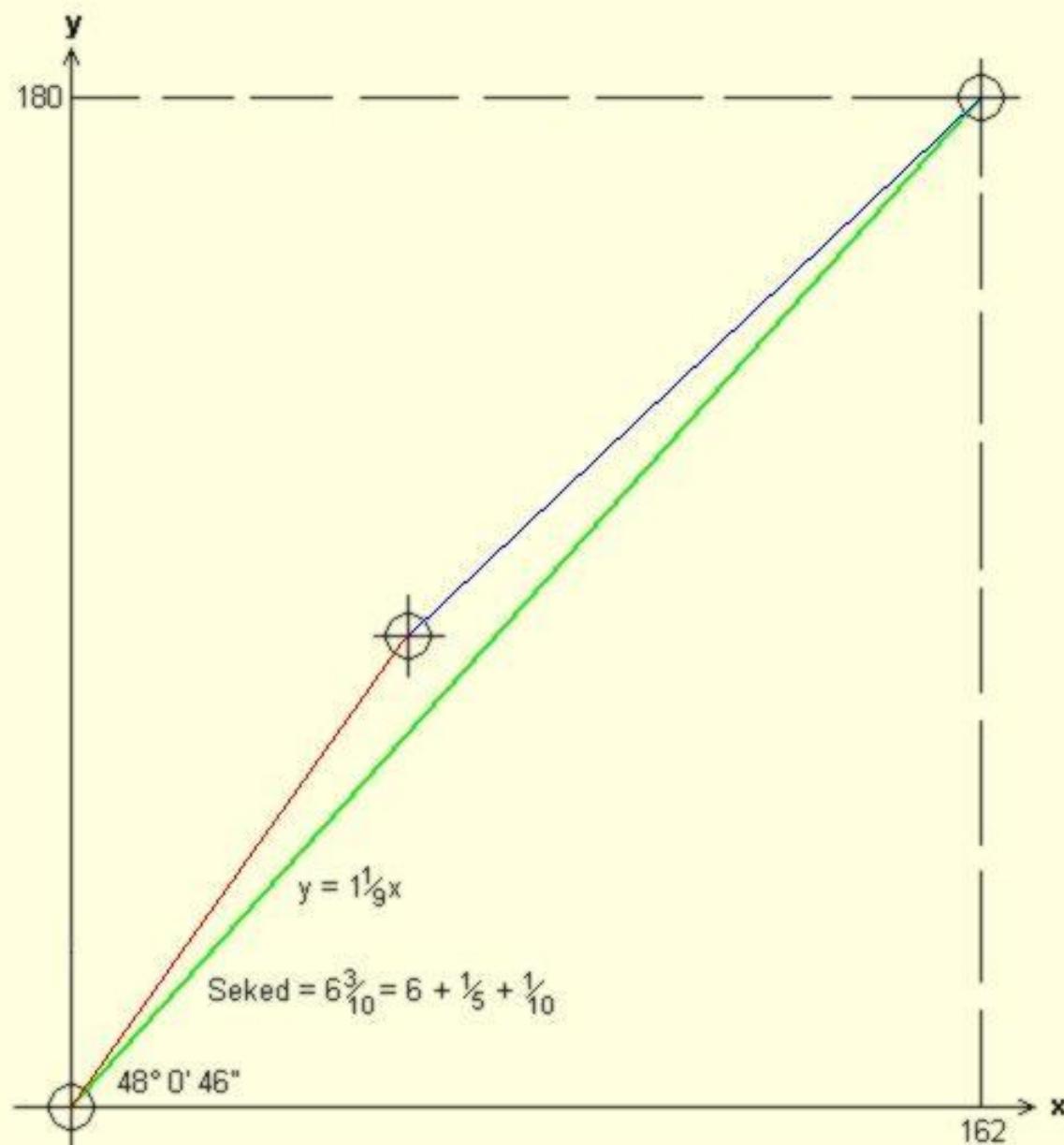


Figure 3-14. Theoretical Slopes of The Southern Shining Pyramid

The theoretical volume of the Southern Shining Pyramid is exactly 7,286,976 cubic Southern Shining cubits.

$$7,286,976 = 12^3 \times 4217$$

The outer form contains an implied inner, or hidden, pyramid.



Unit: Southern Shining cubit = 23.01 English inches

HIDDEN PYRAMID

PWS-201

Figure 3-15. Theoretical Slope of The Hidden Pyramid

The theoretical volume of the Hidden Pyramid is exactly 6,298,560 cubic Southern Shining cubits.

$$6,298,560 = 12^3 \times 9^3 \times 5 = 5 \times 108^3$$

The length of each edge of the 5 cubes so defined is equal to three fifths (0.6) the height of, and to one third the width of, the Hidden Pyramid.

3-6. The Rising Sun Pyramid

The Rising Sun Pyramid, also known as the Red Pyramid because of the colour of its exposed rocky core, is located in the Dahshur pyramid field at $29^{\circ} 48' 30.9''$ N \times $31^{\circ} 12' 22''$ E. This is a large pyramid with a rock core and, like every pyramid and mastaba, has been stripped of its limestone casing by stone robbers, leading directly to the complete and utter collapses of those many pyramids which had been built with mud brick cores. However, with the cores exposed aerial photography and now satellite imaging reveal that

those of both the Horizon Pyramid and the Rising Sun Pyramid are stellated, something that would have remained undetectable and completely unknown had they retained their unstellated casings. This means only that both pyramids are unquestionably products of the same guild of architects, assuming that no single architect working in isolation could have raised all the stone-cored pyramids given both the magnitude of each project individually and the number of other pyramids with similarly stellated stone cores.



Figure 3-16. The Rising Sun Pyramid

Quantity	Measure
Width	220 metres (722 feet)
Height	105 metres (345 feet)
Dihedral angle	43° 22'

Table 3-3. Reported Measures of The Rising Sun Pyramid [17]

The form of the Rising Sun Pyramid is defined by a circle and an equilateral triangle:-

- The perimeter of the equilateral triangle is equal to the circumference of the circle ($p = 22/7$).
- The height of the pyramid is equal to the radius of the circle.
- The width of the pyramid is equal to one side of the equilateral triangle.

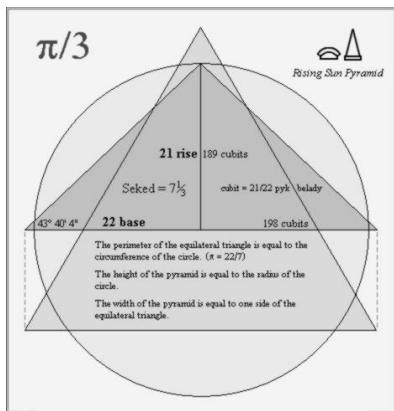


Figure 3-17. Theoretical Geometry of The Rising Sun Pyramid

The circumference of the circle shown in Figure 3-17, equal to the perimeter of the equilateral triangle, is 1,188 ($= 33 \times 6^2$) Rising Sun cubits.

The theoretical volume of the Rising Sun Pyramid is exactly 9,879,408 cubic Rising Sun cubits.

$$9,879,408 = 18^3 \times 11^2 \times 14$$

This volume can be rearranged as a stack of 14 square tiers, each of side 11, of cubes of 18. A single face of that stack is shown below, displaying the proportions of the Horizon Pyramid.

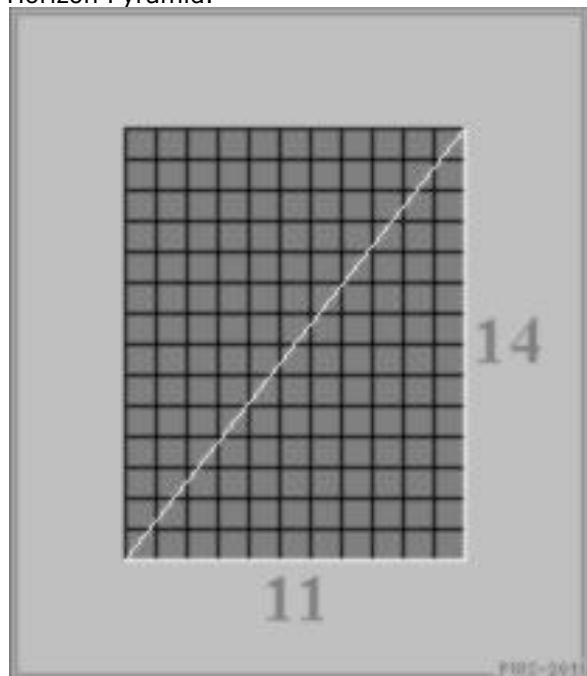


Figure 3-18. Volume Equivalent to The Rising Sun Pyramid

3-7. Theoretical Dimensions of The Stone-Cored Pyramids

Pyramid	Volume	Factors	Unit	Inches
Horizon	13,172,544	$7 \times 33^2 \times 12^3$	Horizon cubit (pyk belady)	22.90
Grand	37,989,750	6×185^3	Horizon foot (2/3 pyk belady)	15.27
Divine	4,143,104	$7 \times 34^2 \times 8^3$	Horizon foot	15.27
Southern Shining (visible)	7,286,976	4217×12^3	Southern shining cubit	23.01
Southern Shining (hidden)	6,298,560	5×108^3	Southern shining cubit	23.01
Southern Shining (visible – hidden)	988,416	$143 \times 2^2 \times 12^3$	Southern shining cubit	23.01
Rising Sun	9,879,408	$14 \times 11^2 \times 18^3$	Rising sun cubit (21/22 pyk belady)	21.86

Table 3-4. Theoretical Volumes of The Stone-Cored Pyramids

To be continued

3-8. Earth And Moon

The proportions of the Horizon Pyramid do indeed relate to the size and shape of the Earth as the legends have it although not, however, in quite the same manner as some have imagined; the proportions of the Horizon Pyramid express the ratio of the diameters of Earth and Moon, as explained below.

Some 33 miles (53km) south of the Horizon Pyramid, at approximately $29^{\circ} 30' N$, is the most northerly latitude where the Moon is ever directly overhead. Since the exact figure for the maximum Obliquity of the Ecliptic has not yet been properly established, depending as it does on observations made over only a few hundred years, where the early observations do not meet modern standards of precision and where the cycle of the Obliquity is currently estimated at some 41,000 years, it is possible that the Moon could, at the maximum of the Obliquity, pass directly overhead the Horizon Pyramid or at the very least overhead its latitude. The last maximum Obliquity took place around 9000 BC and the one before that around 50000 BC.

The radius of the Earth varies between 6,356,752m ($\sim 3,949.9$ miles) at the Poles and 6,378,136m ($\sim 3,963.2$ miles) at the Equator (see [Appendix D](#)). The Moon, on the other hand, is an almost perfect sphere of mean radius 1,738,090.5m (1,080 miles). Hence quotients for Terrestrial radius divided by Lunar radius

range from ~ 3.6573 to ~ 3.6696 . The simplest ratio of whole numbers with a value within this range is 3:11, corresponding to the Terrestrial radius of 6,372,998.5m (3,960 miles) at latitude $29^{\circ}26'45'' N$. Not only is this rather close to the latitude of Giza – about 48 miles (77km) south of it across the surface – but the ratio 3:11 appears in the diagram of the squaring of the circle when the value of π (pi) is taken to be $22/7$. Since the meridional distance from this latitude to that of Giza is less than 0.4% of the Polar circumference, treatment of all three latitudes as identities of the 30th Parallel can be justified. The identification of the latitude of the apex of the Horizon Pyramid with the 30th Parallel is established by their proximity; a mere but exact $0^{\circ}1'15''$ ($1\frac{1}{4}'$), or a 17,280th part of a circle, separates them. Since the meridional width of the base of the Horizon Pyramid is a very precise $7\frac{1}{2}$ arcseconds (from $29^{\circ}58'41.25'' N$ to $29^{\circ}58'48.75'' N$), this translates into a meridional distance apart of almost exactly 10 Horizon Pyramid widths, or 3,960 pyk belady. Clearly the Horizon Pyramid was deliberately located off the 30th Parallel for the purpose of communicating the length of the Egyptian stade to those capable of determining for themselves the exact locations on the face of the Earth of the latitudes involved.

Latitude	Distance below 30th Parallel	Quantity
$30^{\circ} N/S$	Nut	Spherical vesica piscis
$29^{\circ} 58' 45'' N$	75 meridian arcseconds $1\frac{1}{4}$ meridian arcminutes 3,960 pyk belady 10 Horizon Pyramid widths.	Exact latitude of the vertical centreline of the Horizon Pyramid at ground level. This is an exact 1,440th part of the distance from the 30th Parallel to the Equator. The minute of time is an 1,440th part of a day.
$29^{\circ} 48' 30'' N$	690 meridian arcseconds $11\frac{1}{2}$ meridian arcminutes 36,432 pyk belady 92 Horizon Pyramid widths	Assumed intended latitude of the vertical centreline of the Rising Sun Pyramid at ground level (actual = $29^{\circ}48'30.9'' N$)
$29^{\circ} 47' 30'' N$	750 meridian arcseconds $12\frac{1}{2}$ meridian arcminutes 39,600 pyk belady	Assumed intended latitude of the northern precinct wall of the Southern Shining Pyramid (actual = $29^{\circ}47'29.7'' N$) and the vertical centreline of the Black

	100 Horizon Pyramid widths 99 Egyptian stades	Pyramid (estimated). This is an exact 144th part of the distance from the 30th Parallel to the Equator.
29° 45' 30" N/S	870 meridian arcseconds 14½ meridian arcminutes 45,936 pyk belady 116 Horizon Pyramid widths	Exact circumference of parallel = 360 Grand spans = 21,600 English miles or exactly 10 times the long diameter of Moon. arcdegree = Grand span (= 60 English miles - see Figure 3-22) arcminute = English mile arcsecond = 44 English common cubits = 66 corrected English ('Western') feet
~29° 30' N/S	-	Approximate maximum vertex of Moon's orbit at maximum Obliquity of the Ecliptic (last occurrences circa 9000 BC and 50000 BC)
29° 26' 45" N/S	1,995 meridian arcseconds 33¼ meridian arcminutes 105,336 pyk belady 266 Horizon Pyramid widths	Exact ratio of diameters, Earth:Moon = 11:3 Exact radius of Earth = 3,960 English miles = 66 Grand spans Exact radius of Moon (long axis) = 1,080 English miles = 18 Grand spans

Table 3-5. Identities of The 30th Parallel

It should be noted that the recovery here of the ancient Egyptian stade has been made possible only very recently by the emergences of the [Global Positioning System](#) and [Google Earth](#) (see [Appendix C](#)). In other words, with data from artificial satellites and radio telemetry. The question of how the builders could position the Horizon Pyramid so very precisely without such aids belongs with that other great mystery – how the Horizon Pyramid was *built*.

The significance of the 30th Parallel in the scheme of things is that it can be geometrically constructed in cross-section, with compasses and straightedge, via a *vesica piscis*. The three-dimensional analog of the vesica is formed by the overlapping of two equal spheres such that the centre of one lies on the surface of the other, with the 30th Parallel occurring along the intersection of the surfaces.

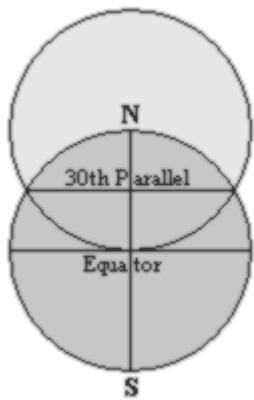


Figure 3-19. Vesica Piscis And 30th Parallel

From here on the latitudes of the apex of the Horizon Pyramid, of the radius of exactly $3\frac{2}{3}$ Lunar radii and of the most northerly overhead Moon will be treated as identities of the 30th Parallel. Furthermore, Moon and Earth will be treated as reference spheres whose radii are in the ratio 3:11. And p will be evaluated at $\frac{22}{7}$, for by doing all of these things a series of remarkable coincidences will be encountered.

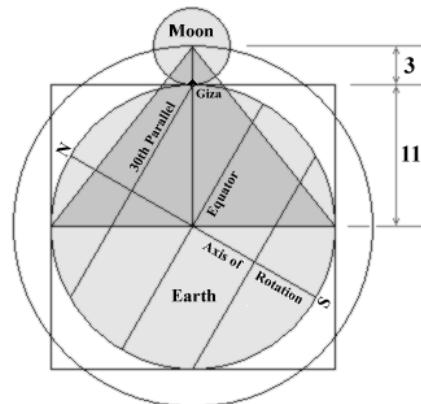


Figure 3-20. Earth, Moon And Horizon Pyramid

Animation 3-1. Earth, Moon And Horizon Pyramid
View: <http://www.odeion.org/atlantis/chapter-3.html>

When the disk of the Moon is also enclosed in a square, the first such coincidence is the appearance of Pythagorean {3-4-5} triangles in the geometrical construction resulting from its juxtaposition with the disk of the Earth.

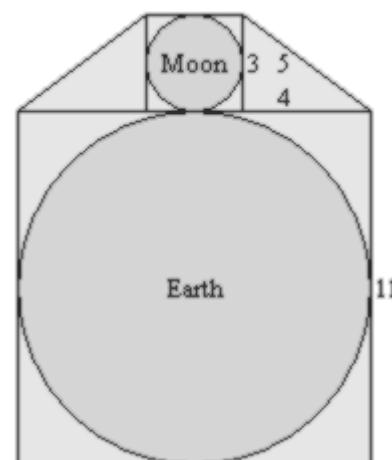


Figure 3-21. Earth, Moon And {3-4-5} Triangles

Extending the construction to form a cross-section of the Grand Pyramid, the lengths of the component lines must be multiplied up, in this case by 12 (the sum of 3, 4 and 5), in order to continue expressing them all as whole numbers. This gives a unit exactly equal to 60 English miles, 60 being the *product* of 3, 4 and 5. Put another way, if the modulus of the diagram shown in Figure 3-22, equal to $1/132^{\text{nd}}$ of the diameter of the Terrestrial reference sphere, the *Grand span*, is subdivided by the product of 3, 4 and 5 then the result is a unit of length which is exactly equal to a modern English or U.S. mile.

In terms of the *Grand span* derived in Figure 3-22, the combined diameters of the Terrestrial and Lunar reference spheres is 168. That is also the number of hours in each and every week. Just as the *Grand span* is exactly divisible into 60 English miles, so the hour is divided into 60 minutes and the number of minutes in

every seven-day period is exactly 10,080. Seven days is very roughly the period of a single *Lunar quarter*.

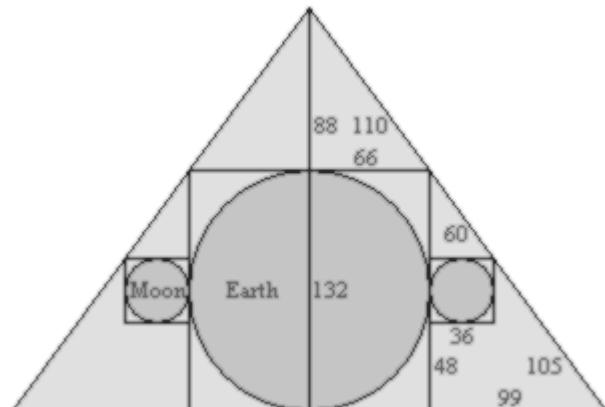


Figure 3-22. Earth, Moon And Grand Pyramid

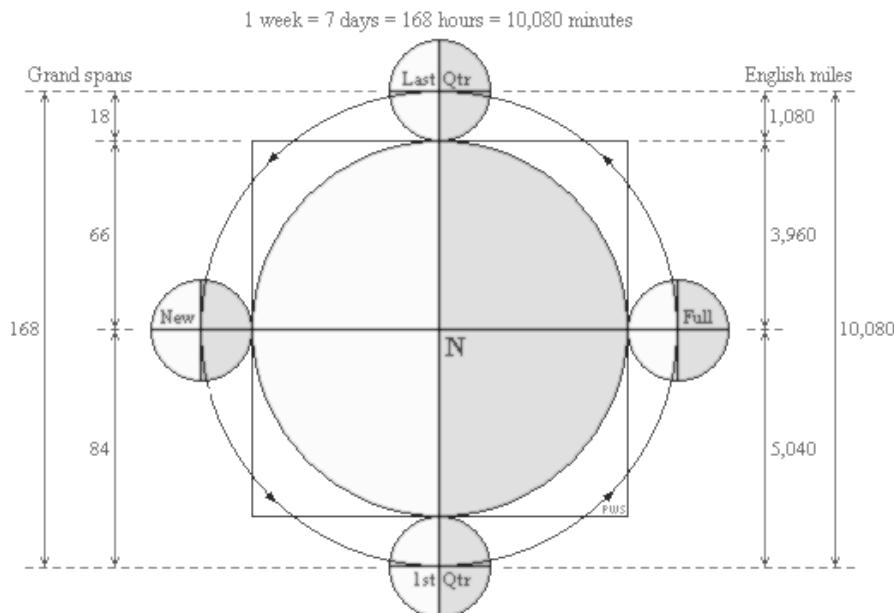


Figure 3-23. The Seven Days of The World

The radius of the large circle whose circumference is equal to the perimeter of the square around the Earth, taking \mathbf{p} (pi) to be $22/7$, is 5,040 miles. 5,040 is the product of all the numbers from 1 to 7, written **7!** and is known as the *factorial of seven*.

5,040 = 1 × 2 × 3 × 4 × 5 × 6 × 7 = 7!

The modulus of the diagram shown in Figure 3-21 is 720 miles. 720 is the factorial of six.

720 = 1 × 2 × 3 × 4 × 5 × 6 = 6!

The circumference of the large circle, again taking $\mathbf{p} = 22/7$, is exactly 528 Grand spans, or 31,680 miles, and this is of course equal in length to the perimeter of the square around the Earth, the base of the virtual cosmic Horizon Pyramid whose height is equal to the combined radii of Earth and Moon, or 5,040 miles.

The story of the *Grand span* does not end there but provides yet another identity of the 30th Parallel. At latitude $29^{\circ}45'30''$ N, some $16\frac{1}{2}$ miles south of the Horizon Pyramid, the circumference of the parallel is exactly 360 Grand spans. This means that the *Grand span* is the length of a degree and the mile the length of a minute of arc along that parallel.

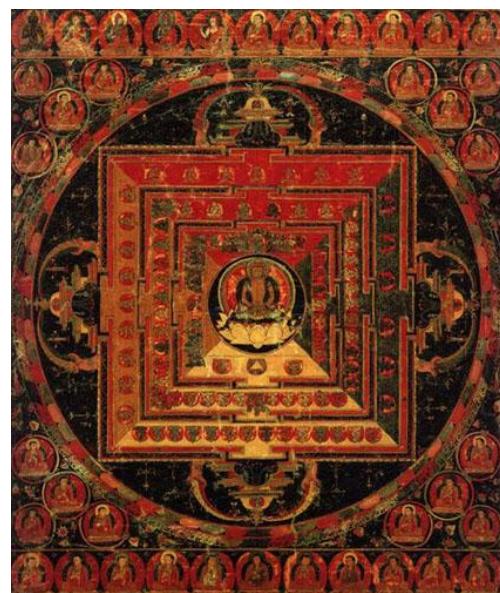


Figure 3-24. Abhisambodhi Vairocana Mandala
(Tibet, 14th Century)

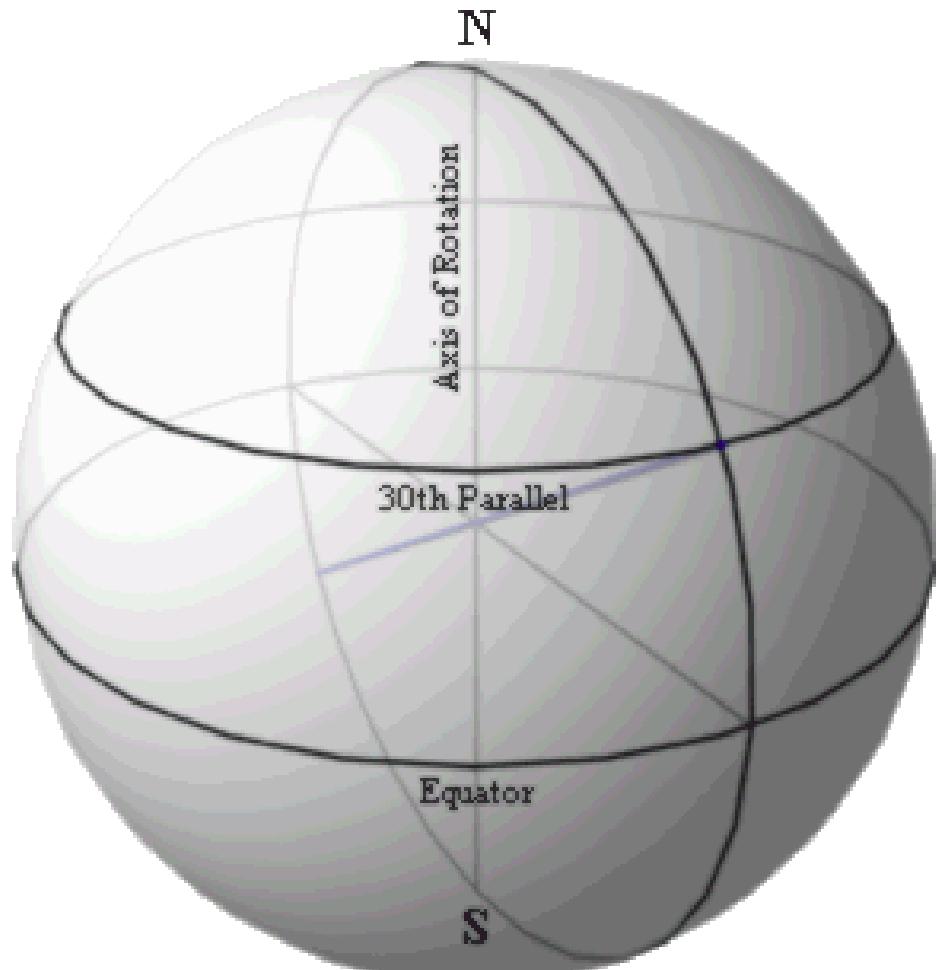


Figure 3-25. The 30th Parallel

3-9. The Western Cubit And Foot

Before the introduction of the Napoleonic system, in one form or another *Long Measure* was in widespread use throughout Western Europe. Evidence for this lies in the Spanish, or *classical*, guitar which will be found always to have a 24-inch *action*, as shown in Figure 3-26.

Twenty four inches or digits comprise a *common cubit*, the particular length here (609.6mm) being the Western European standard, as distinct from Greek, Egyptian or other standardized cubits. The English system includes a *great cubit*, the *yard*, equal to 1½ common cubits, from which the inch is derived*. The English statutory foot, however, appears to be defective or aberrant since it breaks the general rule that makes

the common cubit the harmonic mean of it (the foot) and the great cubit. The foot is that unit in the cubit sequence which cannot be divided in three equal parts each a whole number of digits and which thereby naturally terminates the sequence. Hence a proper *Western foot* would have to be 16 inches (406.4mm) long, just as Greek and Egyptian feet each comprise sixteen of their respective digits.

There are exactly 3,960 such Western feet in a mile, thus the Western foot divides the mile in exactly the same number as the mile divides that radius of the Earth, at $29^{\circ}26'45''$ N, which is in an exact ratio of 11:3 with the radius of the Moon.



Figure 3-27. A 24-Inch Steel Rule (Rabone Chesterman, England, 1970)

* The *Statutum de Admensurazione Terrae* (1305) states [13]:-

"And be it remembered that the Iron Yard of our Lord the King containeth three feet and no more. And a foot ought to contain 12 inches, by the right measure of this yard measured; to wit the thirty-sixth part of this yard rightly measured maketh an inch neither more nor less."

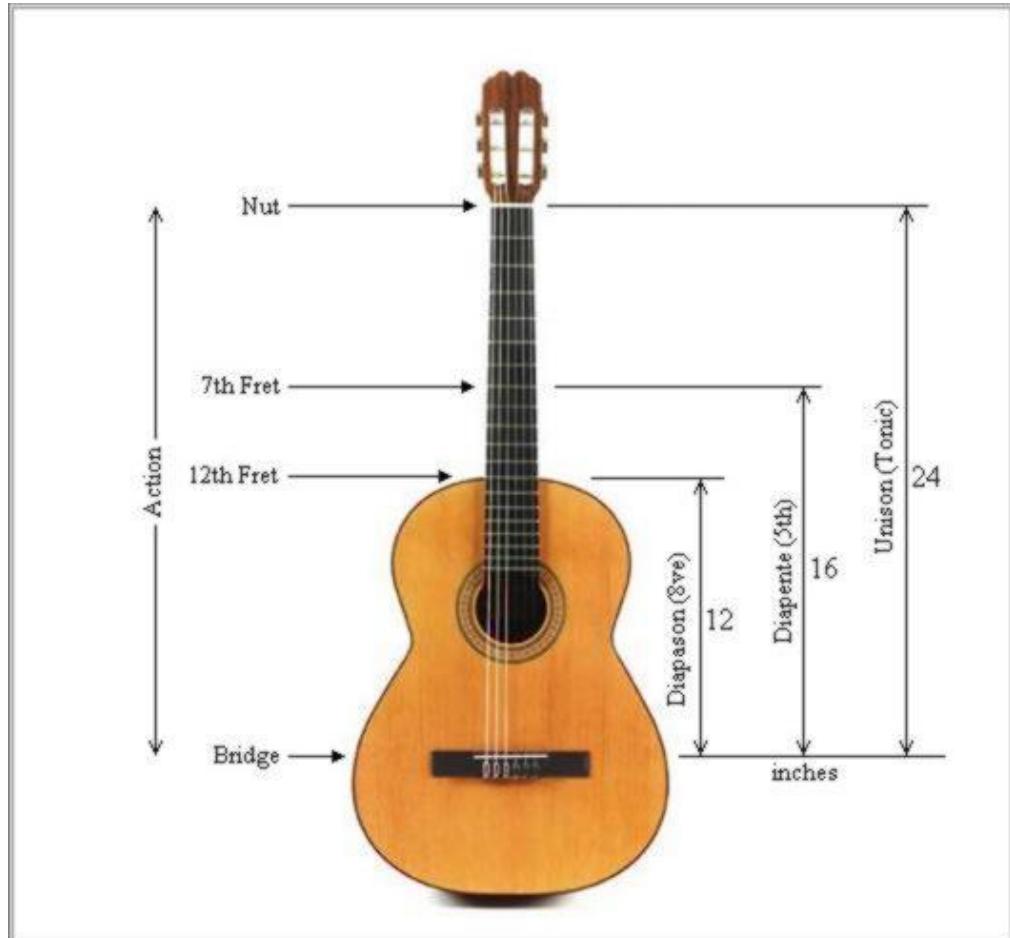


Figure 3-26. The Dimensions of A Classical Guitar

3-10. The Great Circle Way

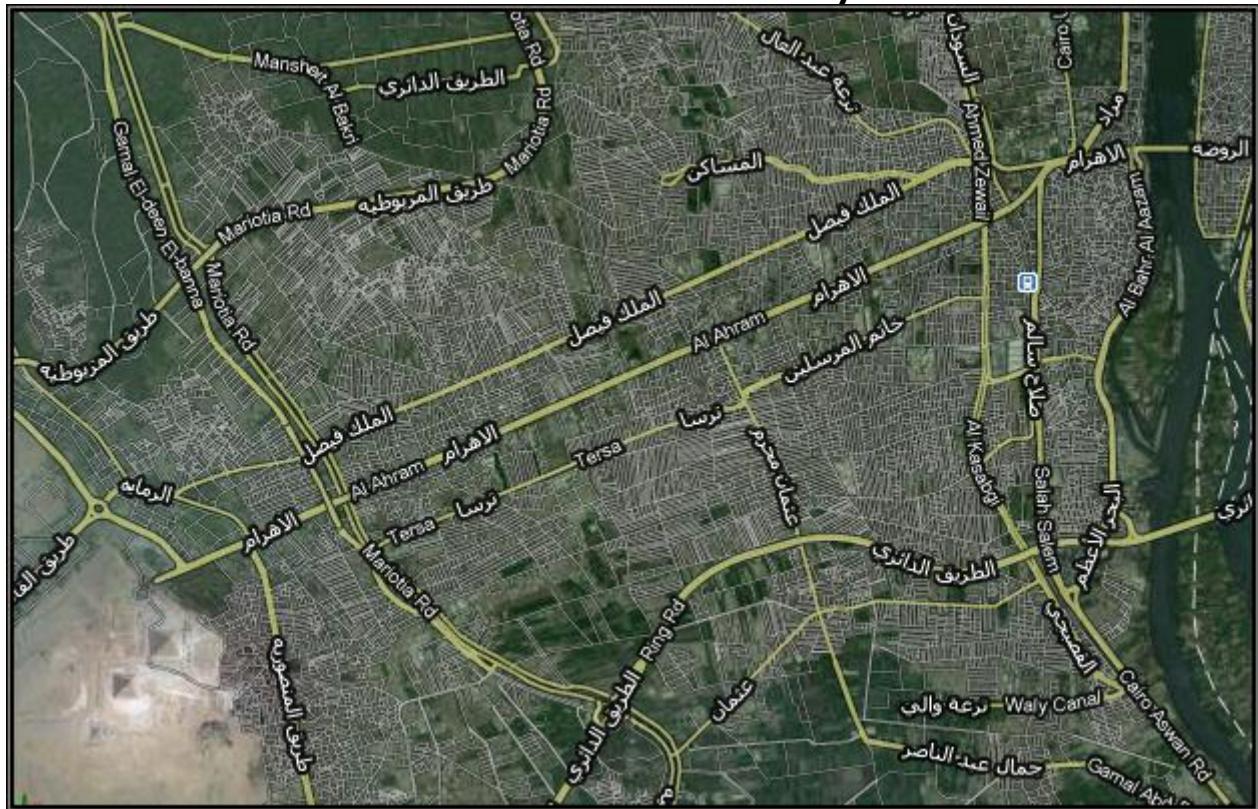


Figure 3-28. Pyramids Road (Map: Google)

Al-Ahram, *Pyramids Road*, is a broad thoroughfare that runs in a straight line WSW from the Nile, terminating in the desert about five hundred yards north of the

Horizon Pyramid. It is very ancient, possibly even more so than the pyramids themselves. It seems as if the city of Cairo itself has sprung up around this very road.

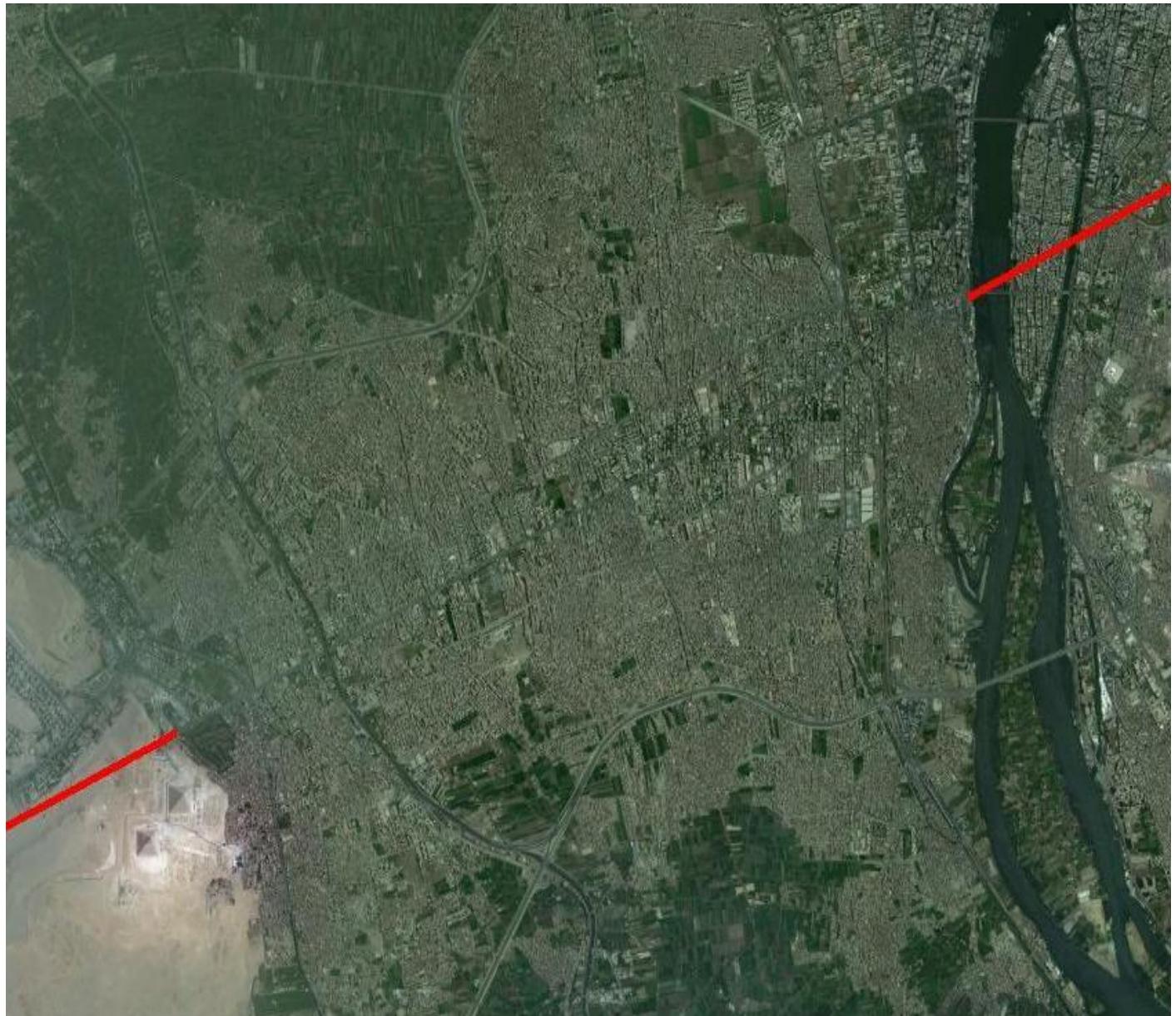


Figure 3-29. The Great Circle Way Discovered in Cairo (*Satellite imagery: Google Earth*)

The azimuth of Pyramids Road at Cairo is 66.666° , making it a segment of a great circle whose northern vertex lies on the High Pass through the Himalayas in the vicinity of Kashmir at coordinates $37^\circ 18' N \times 71^\circ 54' E$. This same great circle is the shortest

uninterrupted land route from the Atlantic coast of Africa, Liberia to be precise, to the Pacific coast of China. Curiously, the point at which it meets the Atlantic is also the eastern end of the shortest route across the Atlantic.

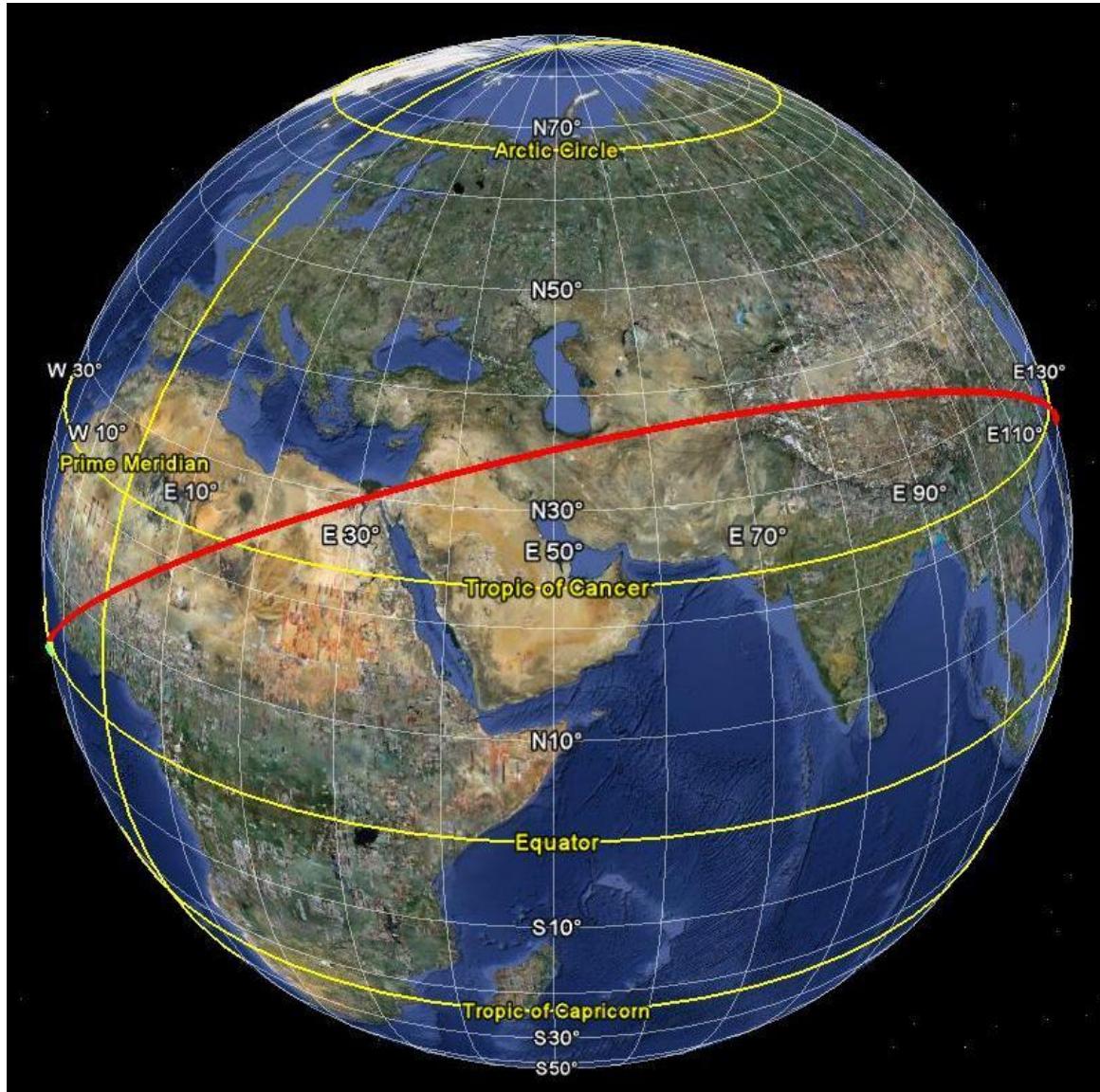


Figure 3-30. The Great Circle Way from Atlantic to Pacific (Satellite imagery: Google Earth)

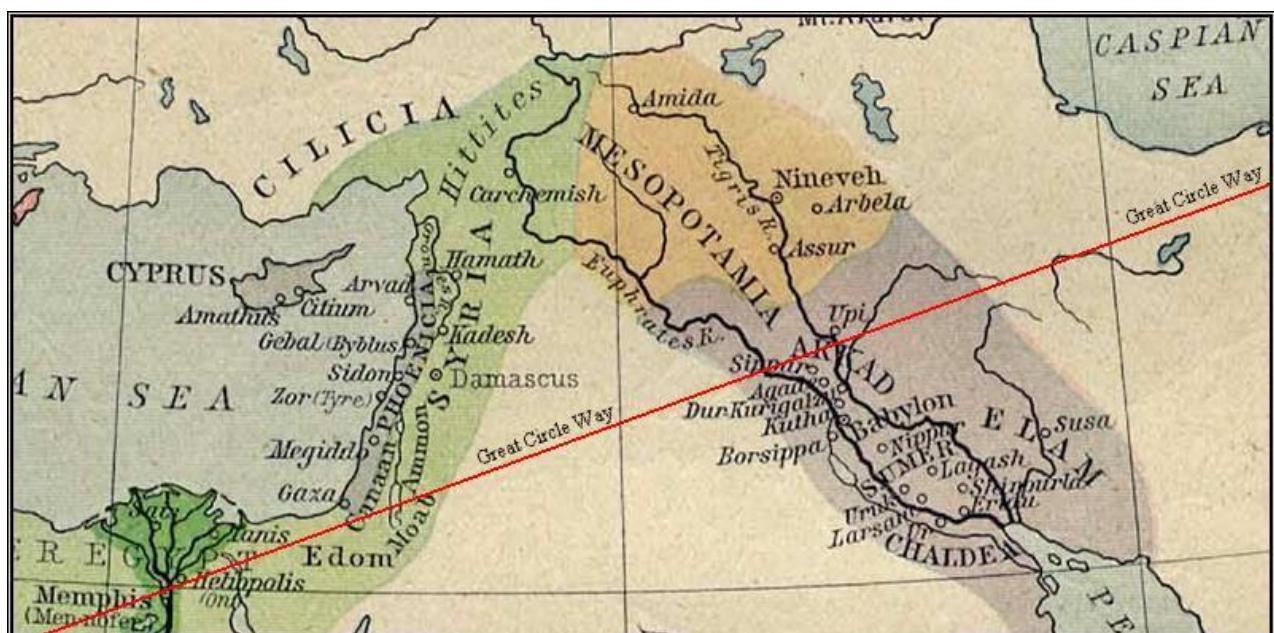


Figure 3-31. The Middle East in 1450 BC Traversed by The Great Circle Way

3-11. The Egyptological Fraud

In 1654 James Usher, Archbishop of Armagh, announced that the Earth was 'created' in 4004 BC. This he did – without a trace of irony – on the basis of a patently absurd Judaic myth. A contemporary of Isaac Newton, John Lightfoot, Vice-chancellor of the University of Cambridge went so far as to narrow it down to 9 o'clock a.m. on the 23rd October of the same year[15]. Moreover, even this year, which must have seemed almost unimaginably ancient to Usher and Lightfoot, was obtained by downright fraud – the presentation of ages measured in months as if they were in solar years. The fraud was neither's but was instead that of the authors of the Judaic mythology, which may not even be a genuine mythology at all but one concocted for political purposes. For example, the age to which Noah is alleged to have lived was 950 solar years and these years are factored into the date of 'Creation' given above. It is quite obvious that the lifespan of Noah was not 950 solar years but the same number of months and using a nominal conversion rate of 13 months per solar year this translates to 73 solar years. Clearly Noah was an ordinary man, alleged extraordinary feats notwithstanding. In correcting this lifespan alone, the date of 'Creation' is brought forward by 877 years. However, Noah was not the only ancient Hebrew credited with an over-extended lifespan. If the age of Adam (930 months = 71½ years) and the period from Adam's death to the birth of Noah (216 months = 16½ years) are similarly corrected, then the date of 'Creation' must be brought forward by 1,935 years. Applying similar corrections to all the "generations of Adam", the year of 'Creation' turns out to have been around 2000 BC at the earliest. If the dubious attribution of the Horizon Pyramid to King Khufu and the even more dubious dating of his reign to around 2600 BC are accepted then the pyramid must have been completed some six hundred years before 'Creation'.

Nonetheless, despite the utter improbability of its claims and the complete lack of any supporting or corroborating evidence of a Hebrew or 'Israelite' presence there, Jean-François Champollion, the so-called 'Father of Egyptology', based his kinglist of Ancient Egypt upon the books of the Torah. As Peter Tompkins puts it:[19]

"By the time he was seventeen, Champollion had made his first table of the succession of Egyptian pharaohs, based on data culled from the Old Testament."

Tompkins displays an interesting turn of phrase. When he states that Champollion "based" his kinglist on the so-called "Old Testament", what he is actually saying (whether he knew it or not) is that Champollion

fabricated a kinglist to conform to the Torah. What that means is that the entire history of the peoples of the Nile Valley as presented to the world by 'Egyptologists' cannot be trusted and is, therefore, worthless. Moreover it means that even if a King Khufu was indeed the commissioner of the Horizon Pyramid, as asserted to Herodotus by the Saite priesthood around 440 BC, the dating of his reign is based upon false evidence. All of the evidence that might have revealed some of the true history has been subject to systematic removal or destruction by subsequent generations of 'Egyptologists' with the singular purpose of continued funding from those with a vested interest in the perpetuation of fabricated histories, in particular that of the Torah. Academia is being selective when it chooses to believe what a priesthood had to say to Herodotus about the commissioner of the Horizon Pyramid and yet to disbelieve that same priesthood when, in around 600 BC, it informed Solon that an entire civilization had disappeared from the face of the Earth some 9,000 years before their time.

Champollion, like Newton before him, was a Freemason and likewise immersed in the masonic machinations of his time.

To be continued

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Now back to the beginning @ **Chapter 1.The Ring of Truth**
Mathematical Derivation of The Musical Intervals

<http://www.odeion.org/atlantis/chapter-1.html>

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5. [Naturally Tempered Frequency Ratios](#)
6. [Correlative Intervals of Just Intonation](#) also available as an [Interactive Computer Program](#)
7. [Lowest Common Denominators of The Ratios of Just Intonation](#)

References and Bibliography

1-1. A Note on Musical Terminology

U.S. readers should give special attention to the usage here of the words *note* and *tone*. In this text, the meanings which they bear derive from the English musical tradition and differ from contemporary U.S. usage. The differences are explained fully in the '[Oxford Companion to Music](#)' to which all those uncertain about these or any other musical terms are referred. Briefly, as used in this book, the word 'note' refers to a pitched sound and to neither the interval of a major step nor to a mark upon a written score. A 'tone' is the interval of a major step, equal to two successive semitones. This definition of the tone does not, however, apply to *combination tones*, also called *resultant tones* and comprising *difference tones* and *summation tones*, all of which belong to the science of *acoustics* and refer to physical vibrations.

the Babylonian unit of rotation, the degree of angle. This analysis, whilst dependent only upon arithmetic calculation, is unavoidably complicated. Those readers who do not wish to tackle it should go straight to [Section 1-5](#), bearing in mind that the conclusion of that section depends upon the arguments laid out in the preceding ones. Anyone wishing to dispute these conclusions must demonstrate that there is a flaw in a preceding section. The author welcomes such challenges; the aim of this book is to establish the truth of the matter and if it is otherwise than stated here then the (false) arguments must necessarily be modified or withdrawn along with any dependent conclusions.

* * *

The musical dimension of interest here is that of *melody* and *harmony* with particular regard to the structure and derivation of the *dodekaphonic* (twelve-notes-per-octave) musical *scale* from which melodies are wrought. The notes of the scale are placed at twelve distinct *intervals*, in the *vibration frequency domain*, from the *root*, or *bass note*, up to and including the *octave* (see Table 1-1 below).

The upwardly delimiting interval is defined by a vibration frequency ratio of 1:2. This is the simplest possible numerical ratio that defines a separation in the vibration-frequency domain (1:1 defines only a point

British	American	Meaning
Note (1)	Tone	Pitched sound
Tone	Note (1)	Major step
Note (2)	Note (2)	Written mark

1-2. Introduction to The Dodekaphonic Intervals

Prior analysis of the derivation of the dodekaphonic scale is a necessary underpinning to the exposition of

location in the frequency continuum) and, for reasons which will be made clear in [Section 1-3](#), is the progenitor of all the intervening intervals.

Classification		Pythagorean Concord	Natural Major Scale	
Pitch	Tonal		Degree	Name
0	Unison		I	Tonic, Root
1	Semitone			
2	Tone		II	Supertonic
3	Minor 3 rd			
4	Major 3 rd		III	Mediant
5	4 th	Diatessaron	IV	Subdominant
6	Tritone			
7	5 th	Diapente	V	Dominant
8	Minor 6 th			
9	Major 6 th		VI	Submediant
10	Minor 7 th			
11	Major 7 th		VII	Leading note
12	Octave	Diapason	VIII	Tonic, Final

Table 1-1. Dodekaphonic Intervals

Key to Table 1-1.

1. Classification-Pitch

Intervals of a scale are measured from the root, or bass, note. The most rational way to index them is in order of increasing separation in the frequency domain. From the tonic to itself is a null interval, hence it is numbered zero; pitch class enumerates intervals as offsets. Pitch class 12 is the upwardly delimiting duple-ratio interval.

2. Classification-Tonal

The traditional names of the intervals are based upon the degrees of the natural major scale (column 4). The octave is so called because it is the eighth note of that scale. The minor intervals are those which do not occur in the major scale.

3. Pythagorean-Concords

The concords are the basis of the Pythagorean theory of harmony. It should be noted that in the table it is the intervals that are named. Pythagoras named the notes corresponding to pitch numbers 0, 5, 7 and 12 as *hypate*, *mese*, *paramese* and *nete* respectively. Thus diatessaron is the interval from hypate to mese, diapente that from hypate to paramese and diapason that from hypate to nete.

4. Natural Major-Degree

The major scale is a seven-note melodic subset of the chromatic scale. It belongs to the class of scale known as *diatonic* which comprise a mix of tone and semitone steps, where each semitone step is separated from the other by at least one intervening tone step. Most simple traditional melodies use only this scale. It is conventional to use Roman numerals for major degrees. The major scale is included here only for comparison with the full dodekaphonic and in explanation of the tonal names of dodekaphonic intervals.

5. Natural Major-Name

Archaic names of the major degrees.

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A musical *note* is a sound in which a given regular vibration predominates to the extent that a listener obtains a clear impression of *pitch*. Although a standard pitch is defined, the actual vibration frequency of this is entirely arbitrary and no more than a convenience for musicians who are to play together and must do so 'in tune' with one another. The vibration of the pitch known as 'Middle A' is fixed by modern convention at a frequency of 440 Hertz (cycles per second). However, the convention has not always been thus and the 'standard' frequency of 'A' has in previous times been higher or lower. Mozart is reputed to have used a piano on which 'A' was tuned to 421.6 Hertz. So-called 'philharmonic pitch' was based upon an 'A' of 452.5 Hertz.

More fundamental than the standardization of any single pitch is that of the *intervals* between different pitches. Where two notes are of the same pitch the resulting interval is that of *unison* ('one sound'). This is not truly an interval and there is really only the one note, no matter how many different simultaneous sources it may have. Additional sources are harmonically redundant; their presence increases only power (experienced as loudness) while introducing neither separation nor contrast. Unison has no extension in the frequency domain.

Although sounding two notes an octave apart creates a separation in the frequency domain, there is no contrast such as is experienced in varying degrees of pungency with all the other intervals. The sensation of an octave interval has a quality analogous to the taste of pure water. This characteristic neutrality, unique to the octave interval, results from the production of a *difference tone* equal to one of the constituents, the lower frequency, or bass note, of the pair. Difference tones were first observed by Giuseppe Tartini (1692-1770) and their frequencies can be calculated by subtracting that of the bass note (the lower of the two frequencies, or *vibration numbers*) from that of the upper note (the higher vibration number). The physical nature of a difference tone is not the same as that of the two vibrations giving rise to it but is regular *amplitude modulation* brought about by the summation of the two constituent vibrations.

...more at

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